Iowa Department of Transportation
Office of Design

Vertical Curve Design

Design Manual
Chapter 2
Alignments

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This section provides information relevant to vertical curves. Vertical curves are used to smooth changes in vertical direction. They fall into one of two categories: crest or sag and can be symmetrical or unsymmetrical. A crest occurs when the arc of the curve is below the VPI. A sag occurs when the arc is above the VPI. Figures 1 and 2 illustrate the components of a vertical curve.

Symmetrical

Typically, vertical curves are symmetrical. This means the tangent length from VPC to VPI equals the tangent length from VPI to VPT. It is not necessary for the VPC and the VPT to be at the same elevation to have a symmetrical vertical curve.

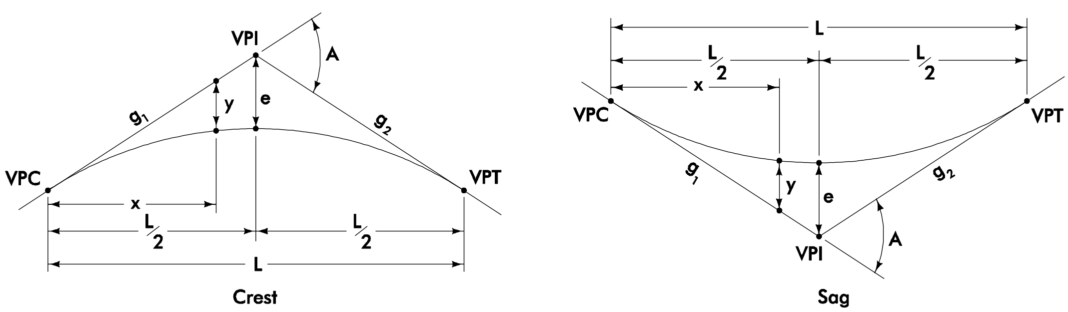

Figure 1: Symmetrical vertical curves.

Unsymmetrical Vertical Curves

There are certain situations when an unsymmetrical vertical curve will better satisfy constraints. An unsymmetrical curve is a curve in which the tangent length from VPC to VPI does not equal the tangent length from VPI to VPT. As already mentioned, symmetrical vertical curves are more common than unsymmetrical vertical curves, but since the designer is likely to encounter both, equations for both are provided.

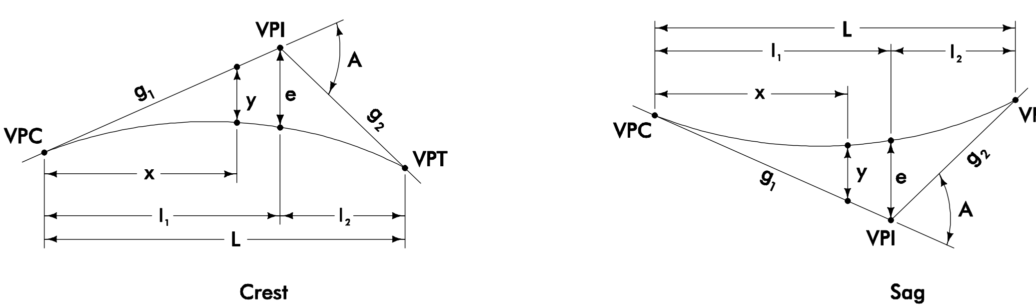

Figure 2: Unsymmetrical vertical curves.

Definitions

Note: g_1 and g_2 are gradients, or tangent grades, of a slope given in percent. These gradients are determined by dividing the difference in elevation of two points by the horizontal distance between them and then multiplying by 100. The definitions below apply to crest and sag vertical curves.

A = Algebraic difference in gradients, $g_2 - g_1$.

L = Total length of vertical curve.

K = Rate of vertical curvature.

l_1 = Length of curve 1 (unsymmetrical vertical curve only).

l_2 = Length of curve 2 (unsymmetrical vertical curve only).

VPC = The Vertical Point of Curvature.

VPT = The Vertical Point of Tangency.

VPI = The Vertical Point of Intersection.

x = Horizontal distance to any point on the curve from the VPC.

x_t = Turning point, which is the minimum or maximum point of the curve.

e = Vertical offset or middle ordinate, which is the vertical distance from the VPI to the arc.

y = Vertical distance at any point on the curve to the tangent grade.

r = Rate of change of grade.

E_{VPC} = Elevation of VPC.

E_{VPT} = Elevation of VPT.

E_x = Elevation of a point on the curve at a distance x from the VPC.

E_t = Elevation of the turning point.

Formulas

Symmetrical Vertical Curves

$A = g_2 - g_1$	g_1 and g_2 are in percent
$K = \dfrac{L}{A}$	L is given in feet (meters) and g_1 and g_2 are in percent
$r = \dfrac{A}{100L}$	L is given in feet (meters) and g_1 and g_2 are in percent
$e = \dfrac{AL}{800}$	L is given in feet (meters)
$y = \dfrac{4ex^2}{L^2} = \dfrac{1}{2}rx^2 = \dfrac{Ax^2}{200L}$	measured from the tangent that passes through VPC and VPI: L and x are given in feet (meters)
$E_x = E_{VPC} + g_1 x + \dfrac{1}{2}rx^2$	L and x are given in feet (meters): convert g_1 to a decimal
$x_t = -\dfrac{g_1}{r}$	x_t is in feet (meters): convert g_1 to a decimal
$E_t = E_{VPC} - \dfrac{g_1^2}{2r}$	convert g_1 to a decimal

Unsymmetrical Curves

$A = g_2 - g_1$	g_2 and g_1 are in percent
$K = \frac{L}{A}$	L is in feet (meters) and g_1 and g_2 are in percent
$r = \frac{A}{100L}$	L is in feet (meters) and g_1 and g_2 are in percent
$r_1 = \frac{A}{100L} * \frac{l_2}{l_1}$	r_1 is the rate of change of grade for the tangent through VPC and VPI: L, l_1, and l_2 are in feet (meters)
$r_2 = \frac{A}{100L} * \frac{l_1}{l_2}$	r_2 is the rate of change of grade for the tangent through VPI and VPT: L, l_1, and l_2 are in feet (meters)
$e = \frac{Al_1l_2}{200L} = \frac{1}{2}r_1l_1^2 = \frac{1}{2}r_2l_2^2$	L, l_1, and l_2 are in feet (meters)
$E_{x1} = E_{VPC} + g_1x + \frac{1}{2}r_1x^2$	calculates elevation on left branch of an unsymmetrical curve, x given in feet (meters): g_1 is converted to a decimal
$E_{x2} = E_{VPT} - g_2x + \frac{1}{2}r_2x^2$	calculates elevation on right branch of an unsymmetrical curve, x given in feet (meters): g_2 is converted to a decimal
$x_{t1} = -\frac{g_1}{r_1}$	if turning point occurs in left branch, where x_{t1} is given in feet (meters): convert g_1 to a decimal
$x_{t2} = -\frac{g_2}{r_2}$	if turning point occurs in right branch, where x_{t1} is given in feet (meters): convert g_2 to a decimal
$E_{t1} = E_{VPC} - \frac{g_1^2}{2r_1}$	if turning point occurs in left branch: convert g_1 to a decimal
$E_{t2} = E_{VPT} - \frac{g_2^2}{2r_2}$	if turning point occurs in right branch: convert g_2 to decimal

Design Considerations

Several items must be considered in the process of designing a vertical curve. Of particular importance are:

- Stopping sight distance for crests and headlight sight distance for sags
- Rate of change of grade
- Drainage
- Appearance

Section 6D-5 of this manual provides additional guidance on the design of vertical curves. AASHTO's *A Policy on Geometric Design of Highways and Streets* provides a comprehensive guide, including a number of useful tables and graphs, to the design of vertical curves. The designer should consult these resources.

Plan Curve Data

The following Plan Curve Data should be provided on the plan sheets for each vertical curve: K, elevation and station of PI, and length of curve.